

# Pion and Kaon Vector Form Factors

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## Abstract

The pion and kaon coupled-channel vector form factors are described by making use of the resonance chiral Lagrangian results together with a suitable unitarization method in order to take care of the final state interactions. A very good reproduction of experimental data is accomplished for the vector form factors up to  $\sqrt{s} \leq 1.2$  GeV and for the  $\pi\pi$  P-wave phase shifts up to  $\sqrt{s} \leq 1.5$  GeV.

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## 1 Unitarization

Using an appropriate unitarization method we take into account the final state interaction corrections to the tree level amplitudes calculated from lowest order  $\chi PT$  [1] and from the inclusion of explicit resonance fields in a chiral symmetry fashion as given in ref. [2]. A similar procedure has already been used in the scalar sector to describe the scalar form factor associated with the strange-change scalar current  $\bar{u}s$  in ref. [3]. Starting from the unitarity of the  $S$ -matrix and the introduction of the electromagnetic meson form factor  $F_{MM'}(s)$ :

$$\langle \gamma(q) | T | M(p) M'(p') \rangle = e \epsilon_\mu (p - p')^\mu F_{MM'}(s) \quad (1)$$

with  $q^2 = s$ ,  $e$  the modulus of the electron charge and  $\epsilon_\mu$  the photon polarization vector, we arrive at the expression:

$$\text{Im} F_{MM'}(s) = \sum_\alpha F_\alpha^*(s) \frac{p_\alpha(s)}{8\pi\sqrt{s}} \theta(s - 4m_\alpha^2) p_\alpha(s) \frac{T(s)_{\alpha, MM'}}{p_{MM'}(s)} \quad (2)$$

where  $\theta(x)$  is the usual Heaviside function. On the other hand  $p_{MM'}(s)$  and  $p_\alpha(s)$  are respectively the moduli of the three momenta of the mesons in the final and intermediate meson states, and we sum over intermediate two-meson states.

We will work in the isospin limit, with  $|\pi\pi\rangle$  and  $|K\bar{K}\rangle$  states (and the  $\rho$  resonance) in the  $I = 1$  channel and only the  $|K\bar{K}\rangle$  state (and the  $\omega$  and  $\phi$  resonances) in the  $I=0$  channel. Using a matrix notation, the P-wave amplitudes  $T^I(s)$  can be written as [4]:

$$T^I = [1 + K^I(s) \cdot g(s)]^{-1} \cdot K^I(s) \quad (3)$$

where  $K_{ij}^I(s)$  are the tree level amplitudes derived from lowest order  $\chi PT$  plus s-channel vector resonance exchange contributions [2] corresponding to the transition  $i \rightarrow j$ . From eqs. (2) and (3) it follows, after some algebra, that  $F^I(s)$  can be written as:

$$F^I(s) = \left[ 1 + \tilde{Q}(s)^{-1} \cdot K^I(s) \cdot \tilde{Q}(s) \cdot g^I(s) \right]^{-1} \cdot R^I(s) \quad (4)$$

where  $\tilde{Q}_{ij}(s) = p_i(s)\delta_{ij}$  and  $K^I(s)$  is the matrix collecting the tree level amplitudes between definite  $\pi\pi$  and  $K\bar{K}$  isospin states.  $F^I(s)$  is the column matrix  $F^I(s)_i = F_i^I(s)$ ,  $R^I(s)$  is a vector made up by functions without any cut and  $g^I(s)$  is the diagonal matrix given by the loop with two meson propagators:

$$g_i^I(s) = \frac{1}{16\pi^2} \left[ -2 + d_i^I + \sigma_i(s) \log \frac{\sigma_i(s) + 1}{\sigma_i(s) - 1} \right] \quad (5)$$

where  $\sigma_i(s) = \sqrt{1 - 4m_i^2/s}$ . We will label pions with 1 and kaons with 2 in the  $I = 1$  case. In the  $I = 0$  case we only have kaons.

In the large  $N_c$  limit loop physics is suppressed and then  $F^I(s) = R_{N_c \text{leading}}(s) = F_t^I(s)$ , where  $F_t^I(s)$  is the tree level form factor.<sup>1</sup>

This allows us to write:

$$F^I(s) = \left[ 1 + \tilde{Q}(s)^{-1} \cdot K^I(s) \cdot \tilde{Q}(s) \cdot g^I(s) \right]^{-1} \cdot [F_t^I(s) + R_{\text{subleading}}^I(s)] \quad (6)$$

with  $R_{\text{subleading}}^I(s)$  being of  $\mathcal{O}(N_c^{-1})$ . If we require that the vector form factor from eq. (6) vanishes for  $s \rightarrow \infty$  as is suggested by the experiments, we find that the subleading part of  $R^I(s)$ , which at first can be an arbitrary polynomial (the poles coming from the resonances are in the leading part  $F_t^I(s)$ ), must be a constant. In order to fix the constants  $R_{\text{subleading}}^I(s)$  and  $d_i^I(s)$  of  $g_i^I(s)$  we match our results with those of one loop  $\chi PT$ . We take  $R_{\text{subleading}}^{I=1}(s) = 0$  in order to constrain further our approach. This can be done since we can match our results with one loop  $\chi PT$  by choosing appropriate values for  $d_1^{I=1}$  and  $d_2^{I=1}$ . The values of the other constants given by the matching are:

$$\begin{aligned} d_1^{I=1} &= \frac{m_K^2}{m_K^2 - m_\pi^2} \left( \log \frac{m_\pi^2}{\mu^2} + \frac{1}{2} \log \frac{m_K^2}{\mu^2} + \frac{1}{2} \right) \\ d_2^{I=1} &= \frac{-2 m_\pi^2}{m_K^2 - m_\pi^2} \left( \log \frac{m_\pi^2}{\mu^2} + \frac{1}{2} \log \frac{m_K^2}{\mu^2} + \frac{1}{2} \right) \\ d^{I=0} &= \frac{1}{3} + \log \frac{m_K^2}{\mu^2} \\ R_{\text{subleading}}^{I=0} &= -\frac{m_K^2}{16\sqrt{2}\pi^2 f^2} \left( \frac{1}{3} + \log \frac{m_K^2}{\mu^2} \right) \end{aligned} \quad (7)$$

The bare masses of the resonances (which appear in the tree level quantities) are fixed by the requirements that the moduli of the  $\pi\pi$   $I = 1$  and  $K\bar{K}$   $I=0$  P-wave amplitudes

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<sup>1</sup>We evaluate the tree level form factors and scattering amplitudes using the lowest order  $\chi PT$  Lagrangian [1] plus the chiral resonance Lagrangian [2].

have a maximum for  $\sqrt{s} = M_\rho^{physical}$  MeV and for  $\sqrt{s} = M_\phi^{physical}$  MeV, respectively. For the mass of the  $\omega$  we take directly 782 MeV since there are no experimental data in the region of the  $\omega$  and its contributions to other physical regions do not depend on such fine details since the  $\omega$  is very narrow. On the other hand, the coupling of the vector resonances [2] to mesons and photons are described by two real parameters  $G_V$  and  $F_V$  respectively. We use their experimental value,  $G_V=53$  MeV (from a study of the pion electromagnetic radii [1]) and  $F_V=154$  MeV (from the observed decay rate  $\Gamma(\rho^0 \rightarrow e^+e^-)$  [2]).

## 2 Results and conclusions.

As can be seen in fig. 1, we can describe in a very precise way the vector pion form factor and the P-wave  $\pi\pi$  phase shifts up to about  $s=1.44$  GeV<sup>2</sup> (even for higher energies in the case of phase shifts). For values of  $\sqrt{s}$  higher than 1.2 GeV new effects appear: 1) the presence of more massive resonances,  $\rho'$ ,  $\omega'$ ,  $\phi'$ ... 2) The effect due to multiparticle states, e.g.  $4\pi$ ,  $\omega\pi$  ... which are non negligible. In figure 2, we compare our results with those of  $\chi PT$ . We can see that the resummation of our scheme leads to a much better agreement with the two loop  $\chi PT$  pion vector form factor than with the one loop one. The resonance regions are also well reproduced.

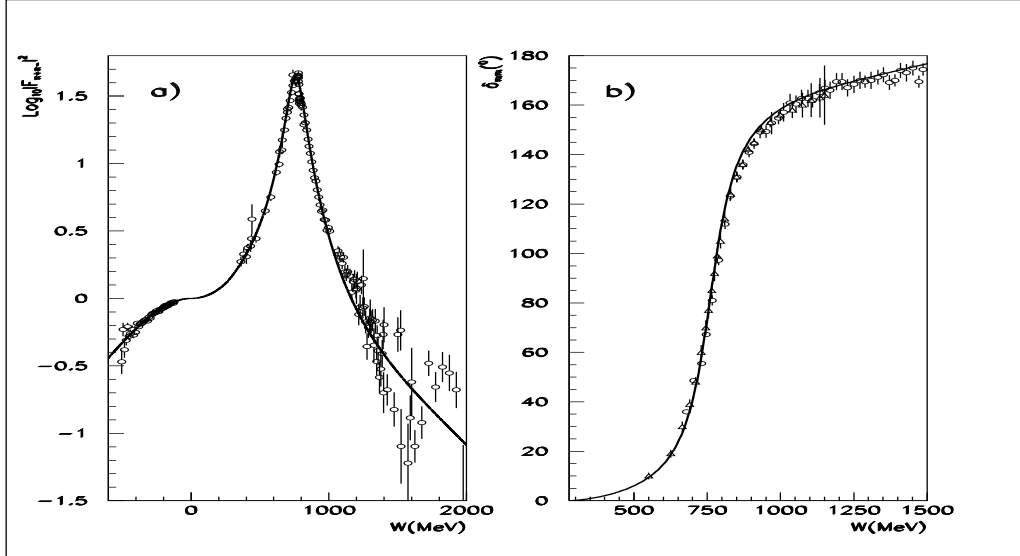


Figure 1:  $W$  is defined as  $\sqrt{s}$  for  $s > 0$  and as  $-\sqrt{-s}$  for  $s < 0$ . a)  $\pi^+\pi^-$  vector form factor. b)  $\pi\pi$  P-wave phase shifts. Both are compared with several experimental data.

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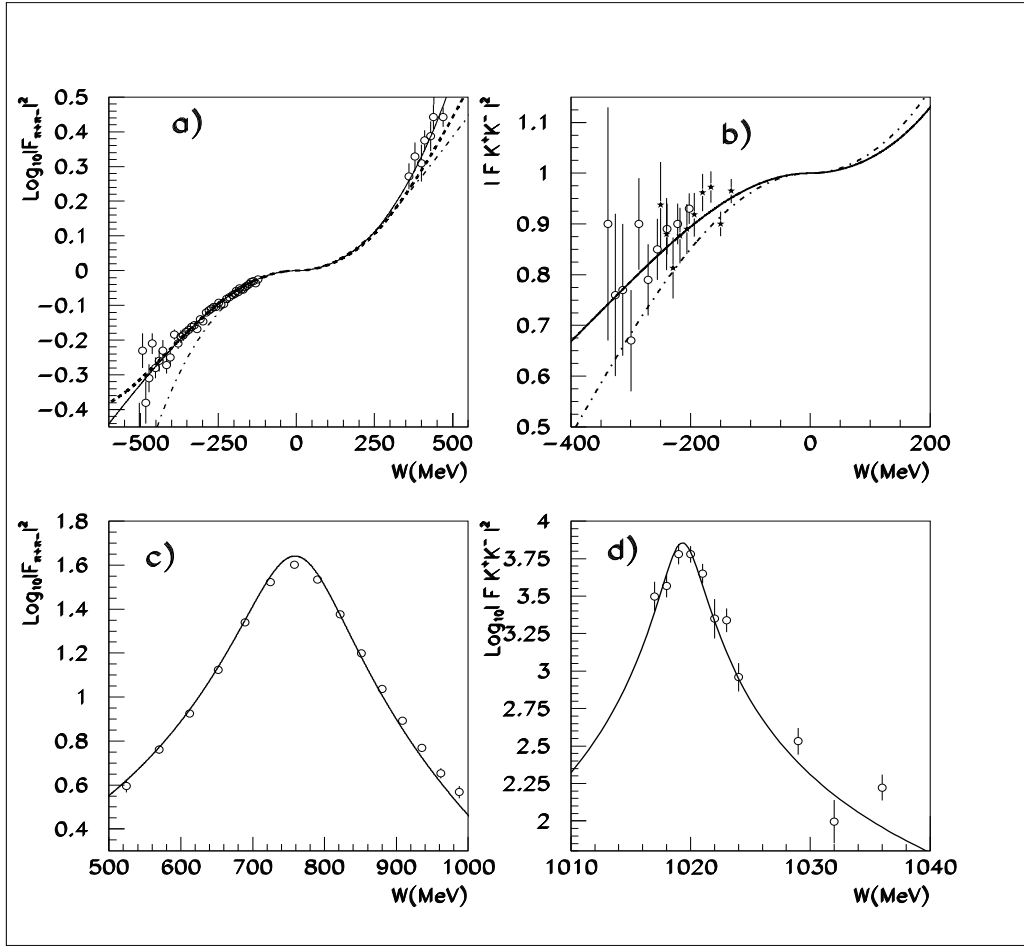


Figure 2:  $W$  is defined as  $\sqrt{s}$  for  $s > 0$  and as  $-\sqrt{-s}$  for  $s < 0$ . From left to right and top to bottom: a) Vector pion form factor. The dashed-dotted line represents one loop  $\chi PT$  ref. [1] and the dashed one the two loop  $\chi PT$  result ref. [5]. b)  $K^+K^-$  form factor. The meaning of the lines is the same as before. c) Vector pion form factor in the  $\rho$  region. Data from tau decay. d)  $K^+K^-$  form factor. All the results are compared with data.

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